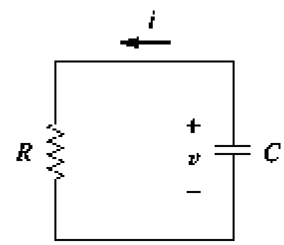
Chapter 4 solved problem

1. In the circuit shown below if

$$v(t) = 56e^{-200t}\ \text{V}, \quad t > 0$$
$$i(t) = 8e^{-200t}\ \text{mA}, \quad t > 0$$

(a) Find the values of *R* and *C*.
(b) Calculate the time constant
(c) Determine the time required for the voltage to decay half its initial value at *t* 0.



Solution

(a) $\tau = RC = 1/200$

For the resistor, $V = iR = 56e^{-200t} = 8Re^{-200t} \times 10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ k\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200X7X10^3} = \underline{0.7143\mu F}$$

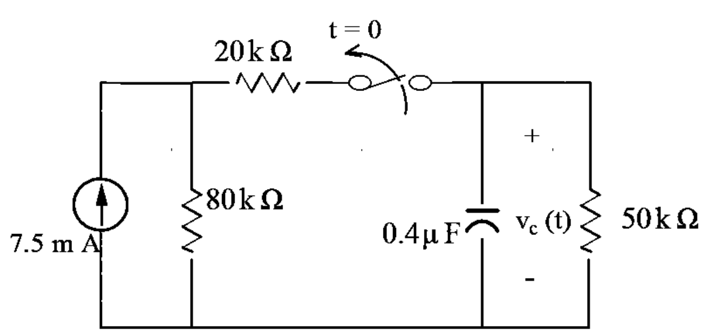
(b) $\tau = 1/200 = 5$ **ms**

(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2}x56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

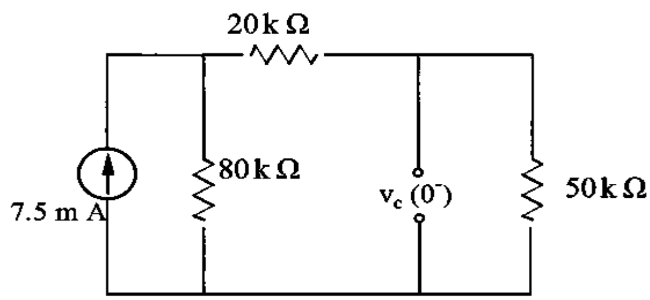
$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ ms}$$

2. Find vc(t) , $t \geq 0$?



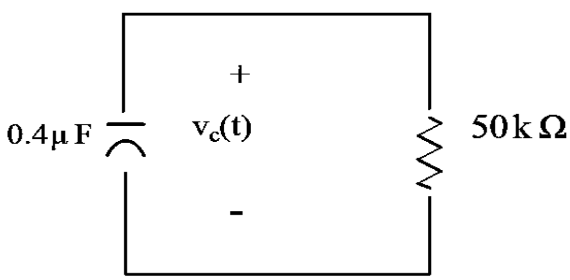
Solution:

- The switch has been closed for long time before switch move to new position.
- The capacitor behaves as open circuit.



$$v_c(0^-) = v_c(0) = v_c(0^+) = 50\,k\Omega\left[(7.5\,mA)\left(\frac{80k}{80k+70k}\right)\right] = 50\,k\Omega\left[(7.5\,mA)\left(\frac{80k}{150k}\right)\right] = 200\ V$$

At t = 0 , the switch is open



$$\frac{dv_c(t)}{dt} + 50\,v_c(t) = 0$$

$$V_c(t) = 200\,e^{-50t}\ \ V.$$

<u>KVL around the loop :</u>

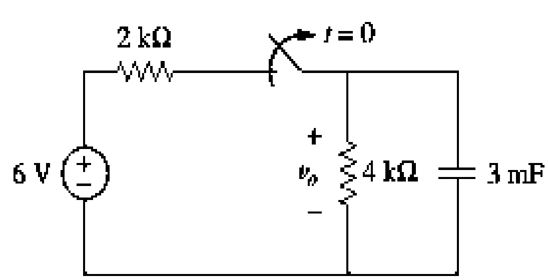
$$v_c(t) + 50k\,i_c(t) = 0$$

$$v_c(t) + 50k\left[C\,\frac{dv_c(t)}{dt}\right] = 0$$

Note From the voltage across the capacitor to find current through capacitor or resistance

I= c(dv/dt)

3. The switch in figure below opens at t=0. Find for Vo for $t > 0$.



Solution **For t < 0, the switch is closed so that**

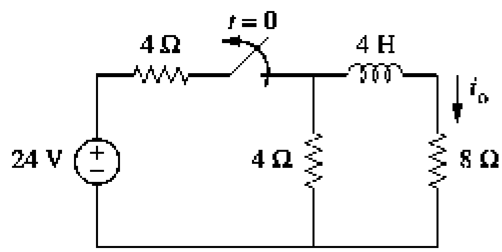
$$v_o(0) = \frac{4}{2+4}(6) = 4 \text{ V}$$

For t >0, we have a source-free RC circuit.

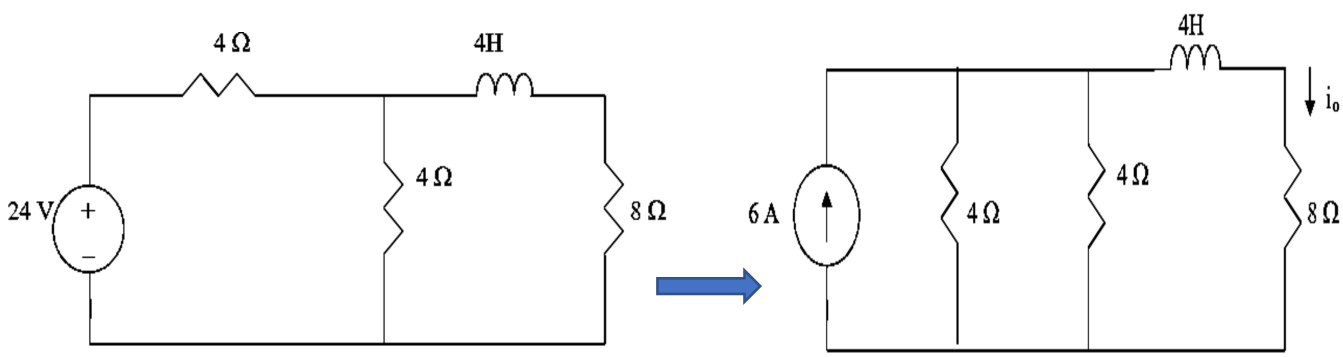
$$\tau = RC = 3x10^{-3} x 4x10^{3} = 12s$$

$$v_o(t) = v_o(0)e^{-t/\tau} = \mathbf{4e^{-t/12}\ V}.$$

4. For the circuit shown in figure below, find for i_o $t > 0$.



Solution: For t<0, we have the circuit shown below.



4||4= 4x4/8=2

$i_o(0^-) = [2/(2+8)]6 = 1.2$ A

For t >0, we have a source-free RL circuit.

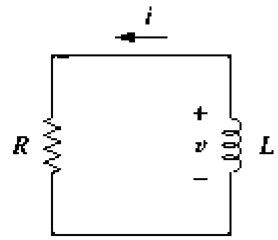
$$\tau = \frac{L}{R} = \frac{4}{4+8} = 1/3 \text{ thus,}$$

$$i_o(t) = \mathbf{1.2e^{-3t}\ A}.$$

5. In the circuit shown below,
   (a) Find *R*, *L*, and. time constant.
   (b) Calculate the energy dissipated in the resistance for $0 < t < .5$ ms.

if $v(t) = 80e^{-10^3 t}\,\text{V}, \quad t > 0$

$i(t) = 5e^{-10^3 t}\,\text{mA}, \quad t > 0$



solution:

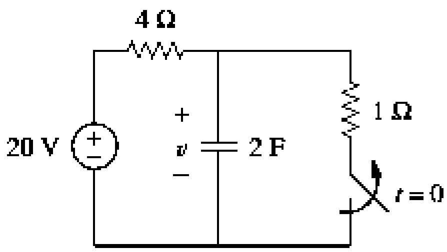
(a) $\tau = \frac{1}{10^3} = \underline{1ms} = \mathbf{1\ ms.}$

$v(t) = i(t)R = 80e^{-1000t}\ V = R5e^{-1000t}x10^{-3}$ or $R = 80{,}000/5 = \mathbf{16\ k\Omega}$.

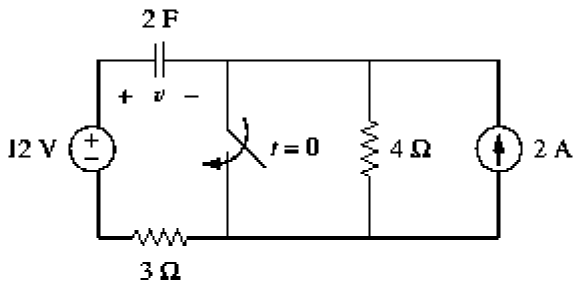
But $\tau = L/R = 1/10^3$ or $L = 16x10^3/10^3 = \mathbf{16\ H}$.

(b). $E = \int_0^{0.05} V(t) * i(t)\,dt$ = 200(1–e–1) x10–6 = **126.42 μJ**.

6. Calculate the capacitor voltage for t < 0 and t > 0 for each of the circuits



(a)



(b)

Solution

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \mathbf{4\ V}$$

After t = 0,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20)\,e^{-t/8}$$

$$v(t) = 20 - 16\,e^{-t/8}\ \text{V}$$

(b) Before $t = 0$, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12 \text{ V}$$

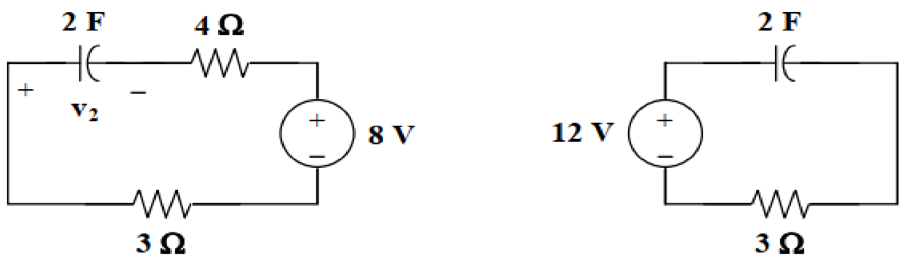
To get v_2, transform the current source as shown in Fig. (a).

$$v_2 = -8 \text{ V}$$

Thus,

$$v = 12 - 8 = \mathbf{4\ V}$$

After $t = 0$, the circuit becomes that shown in Fig. (b).



(a) (b)

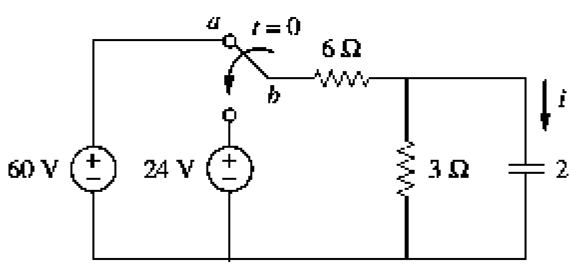
$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12)\, e^{-t/6}$$

$$v(t) = \mathbf{12 - 8e^{-t/6}\ V}$$

7. The switch in Figure below has been in position a for a long time. At t=0 it moves to position b. Calculate i(t) for all t > 0.



Solution:

$$R_{eq} = 6 \,\|\, 3 = 2\ \Omega, \qquad \tau = RC = 4$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

Using voltage division,

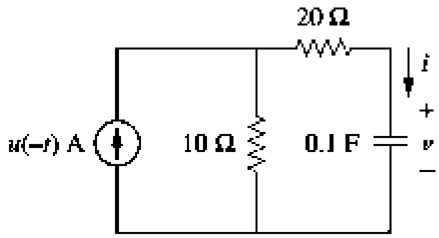
$$v(0) = \frac{3}{3+6}(60) = 20\,V, \qquad v(\infty) = \frac{3}{3+6}(24) = 8\,V$$

Thus,

$$v(t) = 8 + (20-8)\,e^{-t/4} = 8 + 12\,e^{-t/4}$$

$$i(t) = C\frac{dv}{dt} = (2)(12)\left(\frac{-1}{4}\right)e^{-t/4} = -6\,e^{-0.25t}\ A$$

8. Find V(t) and i(t) in the circuit if $u(-t) = 1$,



Solution For t < 0, $u(-t) = 1$,

For t > 0, $u(-t) = 0$, $v(\infty) = 0$

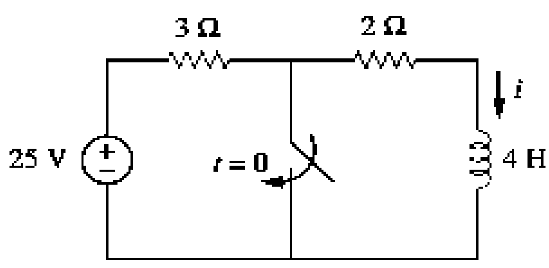
$$R_{th} = 20 + 10 = 30, \qquad \tau = R_{th}C = (30)(0.1) = 3$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty)\right]e^{-t/\tau}$$

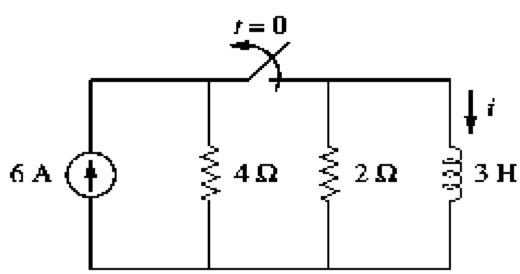
$$v(t) = \mathbf{10\,e^{-t/3}\ V}$$

$$i(t) = C\frac{dv}{dt} = (0.1)\left(\frac{-1}{3}\right)10\,e^{-t/3} \qquad i(t) = \frac{-1}{3}e^{-t/3}\ \mathbf{A}$$

9. Determine the inductor current for both t <0 and t >0 for each of the circuits shown below



(a)



(b)

Solution:

(a) Before t = 0, $i = \frac{25}{3+2} = \mathbf{5\ A}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \qquad i(0) = 5$$

$$i(t) = \boldsymbol{5e^{-t/2}\ u(t)A}$$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

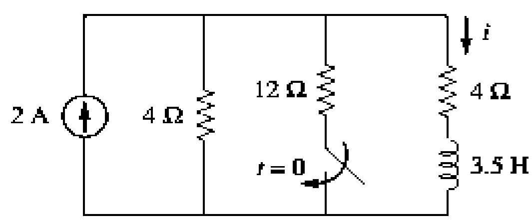
$$i(t) = \mathbf{6\ A}$$

After t = 0, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R} = \frac{3}{2}$$

$$i(t) = \boldsymbol{6e^{-2t/3}\ u(t)A}$$

10. Obtain the inductor current for both t< 0 and t>0 for the circuit shown below



Solution

Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \mathbf{1\ A}$$

After t = 0,

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + (4 \| 12) = 7\ \Omega$$

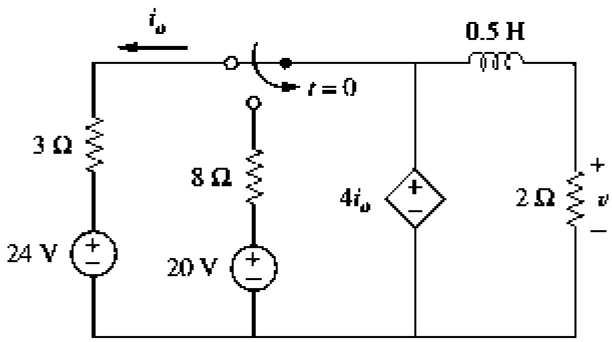
$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{(4\|12)}{4+(4\|12)}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right)e^{-2t}$$

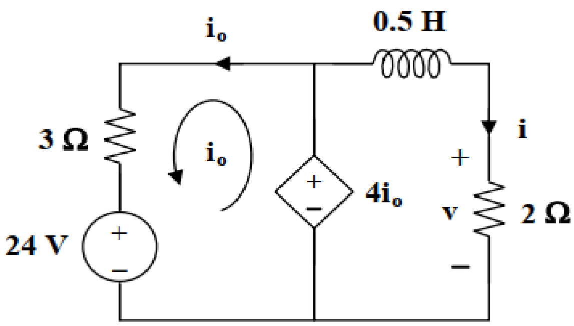
$$i(t) = \mathbf{\frac{1}{7}\left(6 - e^{-2t}\right)\ A}$$

11. Find V(t) for t<0 and t>0 in the circuit shown below

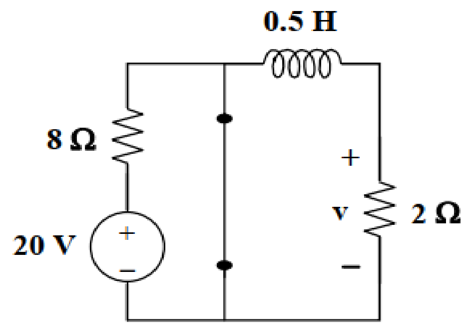


Solution:

For t < 0, consider the circuit shown in Fig. (a).



(a)



(b)

$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$v(t) = 4i_o = \mathbf{96\ V} \qquad i = \frac{v}{2} = 48\ A$$

For t > 0, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

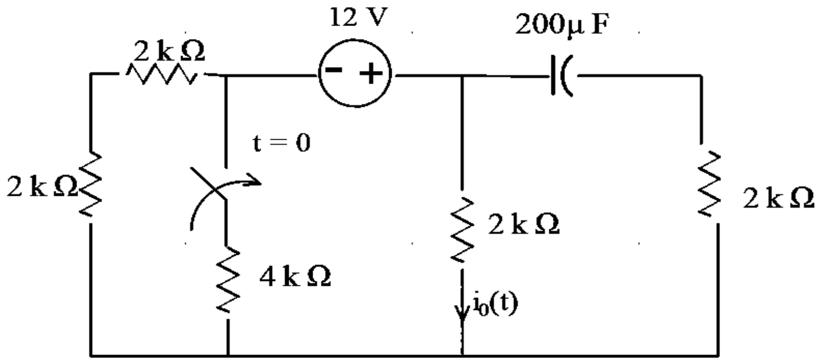
$$i(0) = 48, \qquad i(\infty) = 0$$

$$R_{th} = 2\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{2} = \frac{1}{4}$$

$$i(t) = (48)e^{-4t}$$

$$v(t) = 2i(t) = \boldsymbol{96e^{-4t}\ u(t)V}$$

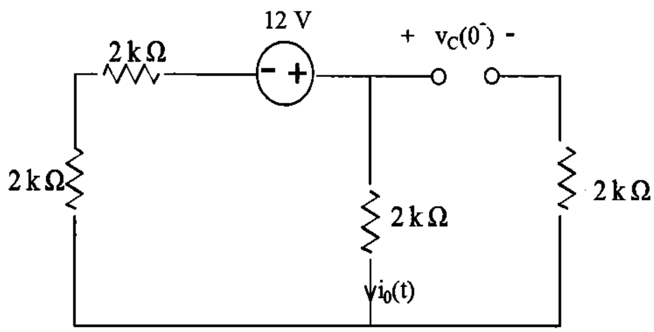
12. Find i0(t) , t > 0 by using step by step methods.



Step (1) :

$$i_0(t) = k_1 + k_2\, e^{\frac{-t}{\tau}}$$

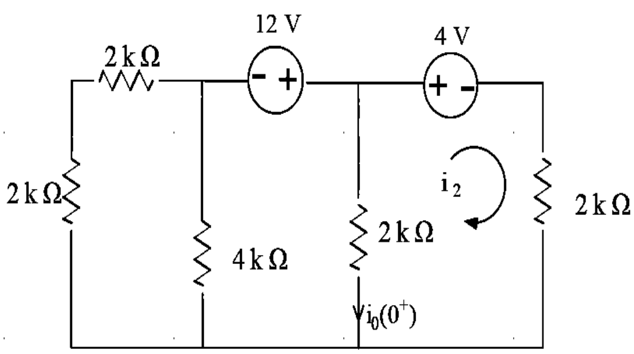
Step (2): assume steady state (for t < 0) replace capacitor by open circuit.

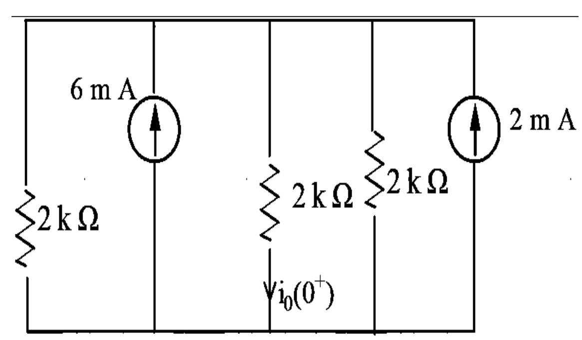


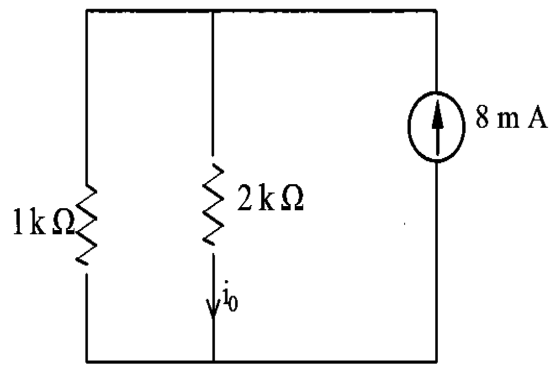
$$v_c(0^-) = (12)\frac{2k}{6k} = 4\text{ V}$$

$$v_c(0^-) = v_c(0) = v_c(0^+) = 4\text{ V}$$

Step 3: now switch is moved, replace capacitor by voltage source = vc(0) ,
Now find i0(0)

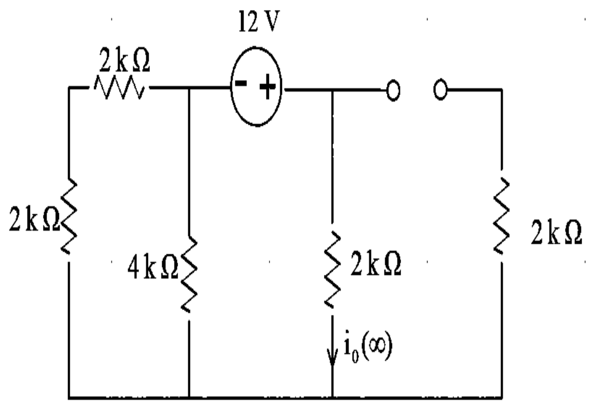


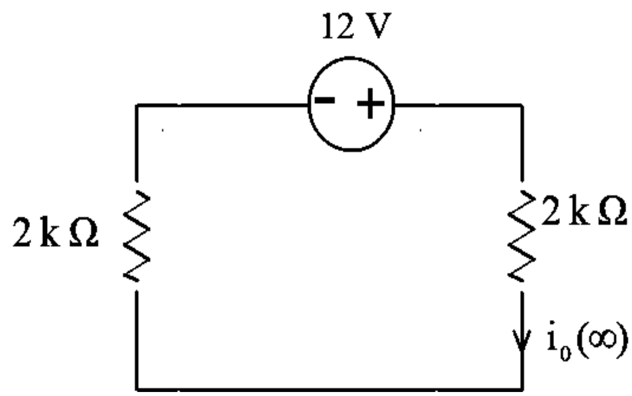




$$i_0(0) = 8\ \text{mA}\left(\frac{1k}{3k}\right) = 2.66\ \text{mA}$$

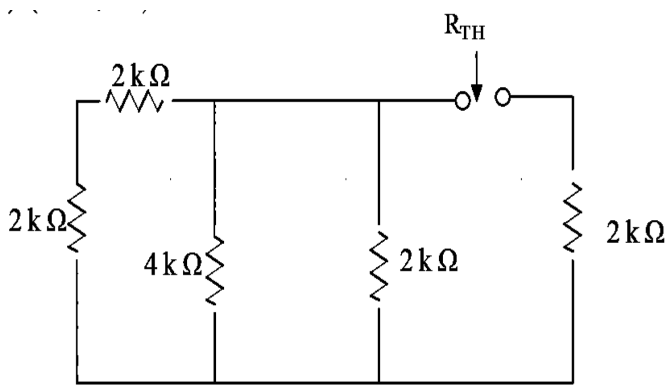
Step 4: assume t = ∞, find $i_0(\infty)$. Steady state Replace capacitor by open circuit





$$i_0(\infty) = \frac{12}{4k} = 3\ \text{mA}$$

Step 5: find time constant. First find RTH at terminals of the capacitor



$$R_{TH} = (4k \,//\, 4k \,//\, 2k) + 2k$$
$$= 3k\Omega$$
$$\tau = R_{TH}\, C = (3k\Omega)(200\mu F) = 0.6\ \text{sec}$$

Step 6 : find the solution $i_0(t)$

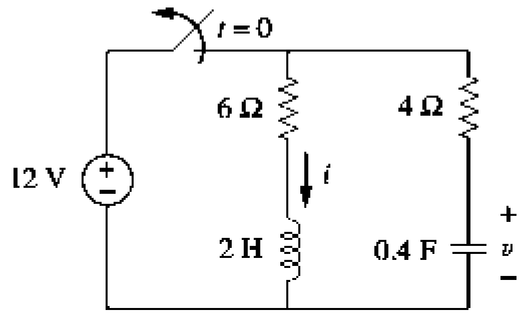
$$i_0(t) = i_0(\infty) + [i_0(0) - i_0(\infty)]\, e^{\frac{-t}{\tau}}$$
$$= 3 + (2.66 - 3)\, e^{-t/0.6}\ \text{mA}$$
$$i_0(t) = 3 - 0.33\, e^{-t/0.6}\ \text{mA}$$

13. For the circuit shown below find

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.



14. If what value of *C* will make an *RLC* series circuit:
   (a) overdamped,
   (b) critically damped,
   (c) underdamped?

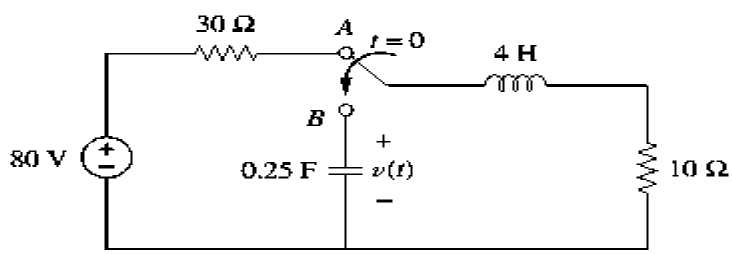
Solution

a. Overdamped when $C > 4L/(R^2) = 4x1.5/2500 = 2.4x10^{-3}$, or C > **2.4 mF**

b. Critically damped when C = **2.4 mF**

c. Underdamped when C < **2.4 mF**

15. The switch in figure below moves from position *A* to position *B* at (please note that the switch must connect to point *B* before it breaks the connection at *A*, a make-before-break switch). Let $v(0) = 0$, find $v(t)$ for $t > 0$.



Solution:

When the switch is in position A, $v(0^-) = 0$ and $i_L(0) = 80/40 = 2$ A. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

When the switch is in position A, $v(0^-) = 0$. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

Since $\alpha > \omega_o$, we have overdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.56\!\!:}\ \ -0.5 \text{ and } -2$$

$$v(t) = Ae^{-2t} + Be^{-0.5t}$$

$$v(0) = 0 = A + B$$

$i_C(0) = C(dv(0)/dt) = -2$ or $dv(0)/dt = -2/C = -8$.

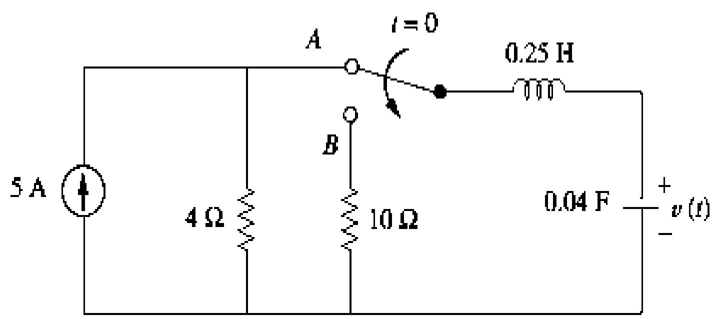
But $$\frac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$$

$$\frac{dv(0)}{dt} = -2A - 0.5B = -8$$

Solving (2) and (3) gives **A**= 1.3333 and **B** = –1.3333

$$v(t) = \mathbf{5.333e^{-2t} - 5.333e^{-0.5t}}\ \mathbf{V}.$$

16. In the circuit of Fig. 8.71, the switch instantaneously moves from position *A* to *B* at Find for all $t > 0$.



Solution:

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x5 = 20$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0 + 20) = -80$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20,\ which is > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1 e^{-2.679t} + A_2 e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2, \quad \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -80$$

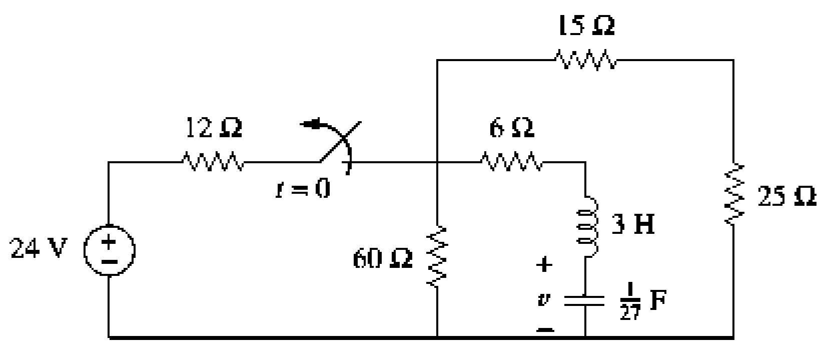
$$\text{This leads to } A_1 = -2.309 = -A_2$$

$$i(t) = 2.309\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$\text{Since, } v(t) = \frac{1}{C}\int_0^t i(t)dt + 20, \text{ we get}$$

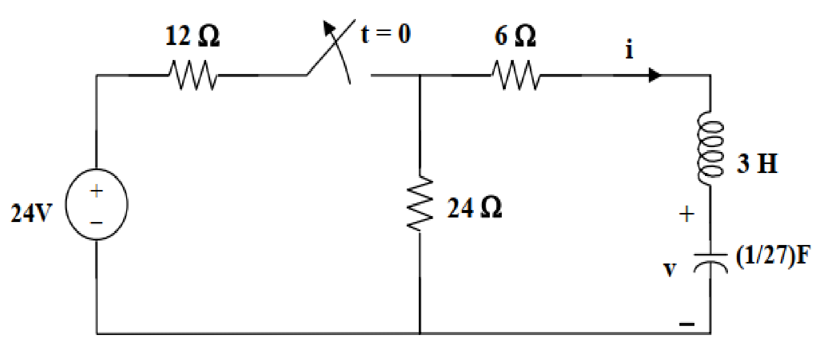
$$\mathbf{v(t) = [21.55e^{-2.679t} - 1.55e^{-37.32t}]\ V}$$

17. Calculate for in the circuit of shown below



Solution: By combining some resistors, the circuit is equivalent to that shown below.

60||(15 + 25) = 24 ohms.



At t = 0-, i(0) = 0, v(0) = 24x24/36 = 16V

For $t > 0$, we have a series RLC circuit. R = 30 ohms, L = 3 H, C = (1/27) F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3x1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}], \quad v(0) = 16 = A + B \quad (1)$$

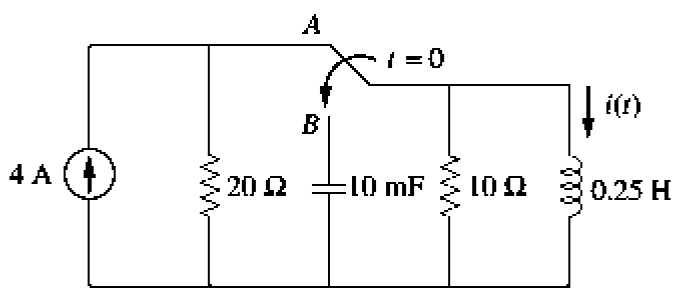
$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A - 9B] \text{ or } A = -9B \quad (2)$$

From (1) and (2), $B = -2$ and $A = 18$.

Hence, $v(t) = (18e^{-t} - 2e^{-9t})$ V

18. The switch in Fig. below moves from position *A* to position *B* at (please note that the switch must connect to point *B* before it breaks the connection at *A*, a make-before-break switch). Determine $i(t)$ for $t > 0$.



Solution:

When the switch is in position A, the inductor acts like a short circuit so $i(0-) = 4$
When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25 - 400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

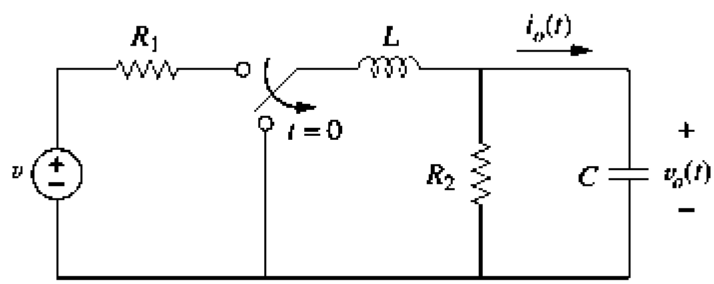
$$v = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}\left(-5A_1\cos 19.365t - 5A_2\sin 19.365t - 19.365A_1\sin 19.365t + 19.365A_2\cos 19.365t\right)$$

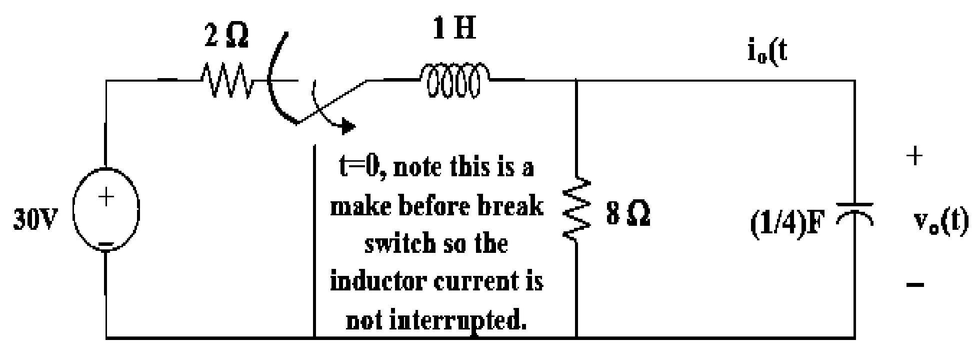
$0 = [di(0)/dt] = -5A_1 + 19.365A_2$ or $A_2 = 20/19.365 = 1.0328$

$$i(t) = e^{-5t}[4\cos(19.365t) + 1.0328\sin(19.365t)]\ \text{A}$$

19. Find Io(t)and Vo(t) for the circuit shown below if v= 30v, L=1H, R1=2Ω, R2=8Ω C=.0.25 F



Solution:



At $t = 0^-$, $v_o(0) = (8/(2 + 8)(30) = 24$

For $t > 0$, we have a source-free parallel RLC circuit.

$\alpha = 1/(2RC) = ¼$ $\qquad \omega_o = 1/\sqrt{LC} = 1/\sqrt{1 \times 1/4} = 2$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4-(1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$$v_o(0) = 30(8/(2+8)) = 24 = A_1 \text{ and } i_o(t) = C(dv_o/dt) = 0 \text{ when } t = 0.$$

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

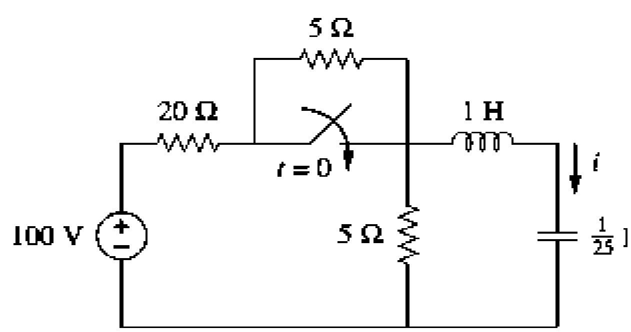
at $t = 0$, we get $dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$$\mathbf{v_o(t) = (24\cos 1.9843t + 3.024\sin 1.9843t)e^{-t/4} \text{ volts.}}$$

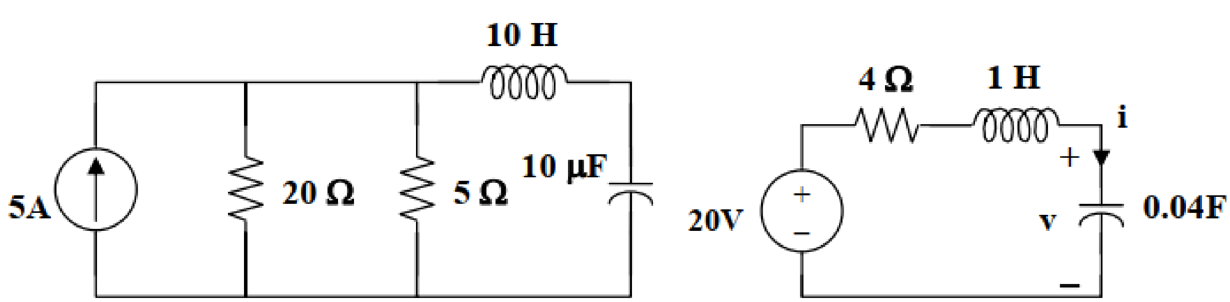
$i_o(t) = Cdv/dt = 0.25[-24(1.9843)\sin 1.9843t + 3.024(1.9843)\cos 1.9843t - 0.25(24\cos 1.9843t) - 0.25(3.024\sin 1.9843t)]e^{-t/4}$
$= \mathbf{[-12.095\sin 1.9843t]e^{-t/4} \text{ A}}$.

20. Find i(t) for t>0 in the circuit of shown below



Solution: At t = 0–, the switch is open. i(0) = 0, and
v(0) = 5x100/(20 + 5 + 5) = 50/3

For t > 0, we have a series RLC circuit shown in Figure (a). After source transformation, it becomes that shown in Figure (b).



(a) (b)

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$

$\alpha = R/(2L) = (4)/(2x1) = 2$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$

Thus, $v(t) = V_s + [(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}]$,

where $\omega_d = 4.583$ and $V_s = 20$

$v(0) = 50/3 = 20 + A$ or $A = -10/3$

$i(t) = Cdv/dt$
$= C(-2)\,[(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}] + C\omega_d[(-A\sin(\omega_d t) + B\cos(\omega_d t))e^{-2t}]$

$i(0) = 0 = -2A + \omega_d B$

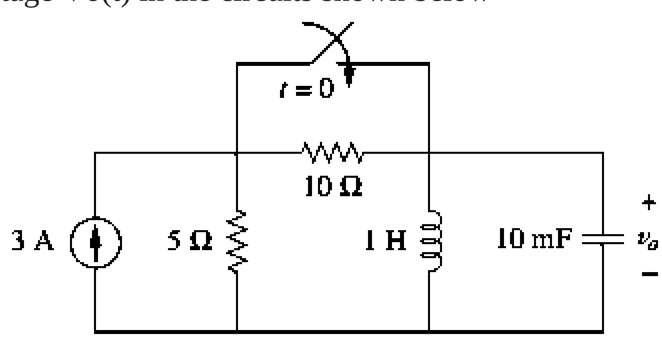
$B = 2A/\omega_d = -20/(3x4.583) = -1.455$

$i(t) = C\{[(0\cos(\omega_d t) + (-2B - \omega_d A)\sin(\omega_d t))]e^{-2t}\}$

$= (1/25)\{[(2.91 + 15.2767)\sin(\omega_d t))]e^{-2t}\}$

$$i(t) = 727.5\sin(4.583t)e^{-2t} \text{ mA}$$

21. Find the output voltage Vo(t) in the circuits shown below



Solution: At t = 0-, we obtain, $i_L(0) = 3x5/(10 + 5) = 1A$ and $v_o(0) = 0$.

For t > 0, the 10-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $$i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$$

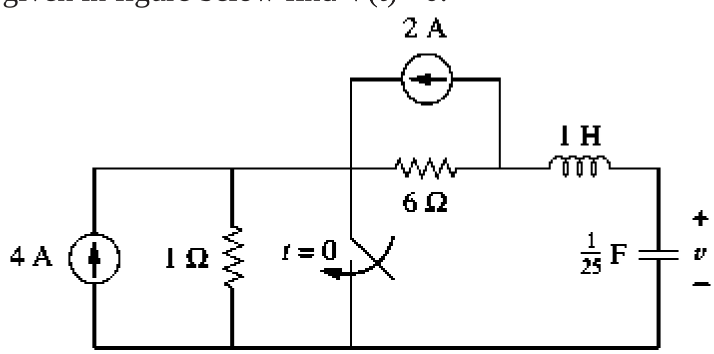
$$i(0) = 1 = 3 + A \text{ or } A = -2$$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -20$$

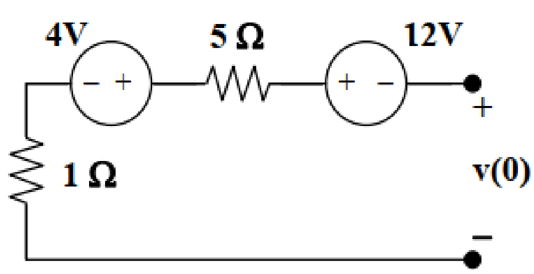
Thus, $v_o(t) = (200te^{-10t})$ V

22. For the network given in figure below find V(t) >0.

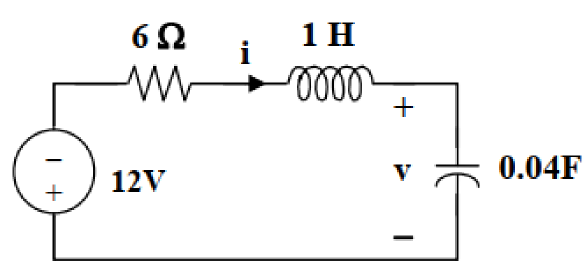


Solution:

For t = 0-, we have the equivalent circuit as shown in Figure (a).
i(0) = i(0-) = 0, and v(0) = 4 – 12 = -8V



(a)



(b)

For $t > 0$, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $$v(t) = V_s + [(A\cos4t + B\sin4t)e^{-3t}], \quad V_s = -12$$

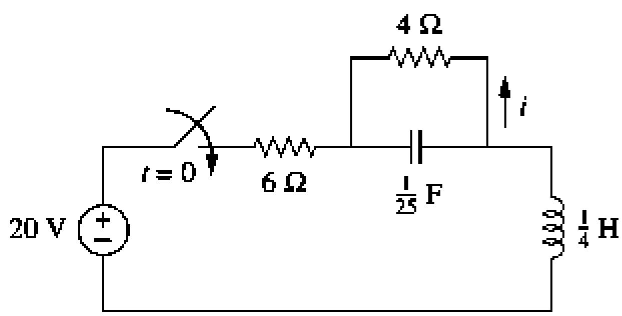
$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$$i = Cdv/dt, \text{ or } i/C = dv/dt = [-3(A\cos4t + B\sin4t)e^{-3t}] + [4(-A\sin4t + B\cos4t)e^{-3t}]$$

$$i(0) = -3A + 4B \text{ or } B = 3$$

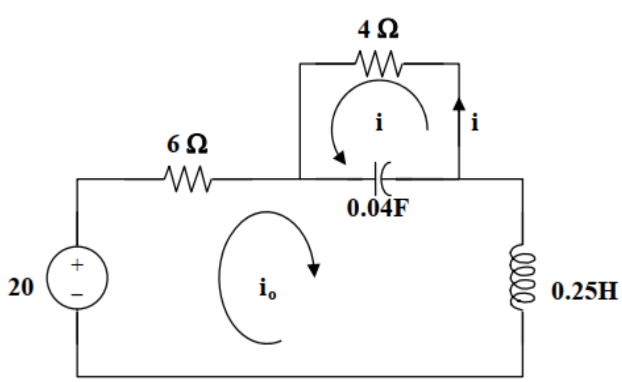
$$\mathbf{v(t) = \{-12 + [(4\cos4t + 3\sin4t)e^{-3t}]\}\ A}$$

23. In the circuit of Figure below, find for $i(t)$ for $t > 0$.



Solution: For t < 0, i(0) = 0 and v(0) = 0.

For t > 0, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int(i_o + i)dt = 0$$

Taking the derivative,

$$6di_o/dt + 0.25d^2i_o/dt^2 + 25(i_o + i) = 0$$

For the smaller loop, $4 + 25\int (i + i_o)dt = 0$

Taking the derivative, $25(i + i_o) = 0$ or $i = -i_o$ (2)

From (1) and (2) $6di_o/dt + 0.25d^2i_o/dt^2 = 0$

This leads to, $0.25s^2 + 6s = 0$ or $s_{1,2} = 0, -24$

$i_o(t) = (A + Be^{-24t})$ and $i_o(0) = 0 = A + B$ or $B = -A$

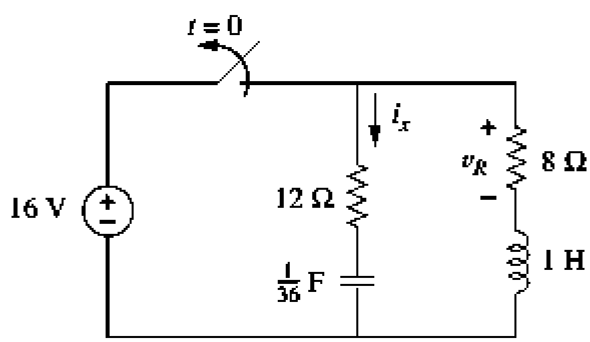
As t approaches infinity, $i_o(\infty) = 20/10 = 2 = A$, therefore $B = -2$

Thus, $i_o(t) = (2 - 2e^{-24t}) = -i(t)$ or

$$i(t) = (-2 + 2e^{-24t})\ A$$

24. If the switch in figure below has been closed for a long time before but is opened at determine: (a) the characteristic equation of the circuit,

(b) i_x and V_R for $t>0$.



Solution: (a) Let v = capacitor voltage and i = inductor current. At t = 0-, the switch is closed and the circuit has reached steady-state.

v(0-) = 16V and i(0-) = 16/8 = 2A
At t = 0+, the switch is open but, v(0+) = 16 and i(0+) = 2.

We now have a source-free RLC circuit. R= 8 + 12 = 20 ohms, L = 1H, C = 4mF

$\alpha = R/(2L) = (20)/(2x1) = 10$

Since $\alpha > \omega_o$, we have a overdamped response.

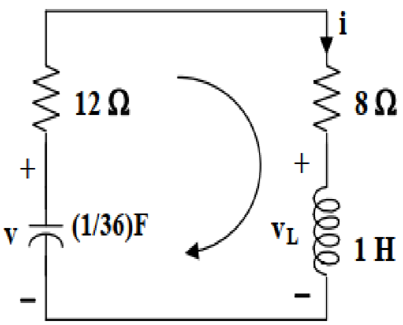
$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x(1/36)} = 6$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -18, -2$$

Thus, the characteristic equation is (s + 2)(s + 18) = 0 or $\mathbf{s^2 + 20s + 36 = 0}$.

(b) $i(t) = [Ae^{-2t} + Be^{-18t}]$ and $i(0) = 2 = A + B$ (1)

To get di(0)/dt, consider the circuit below at t = 0+.



$$-v(0) + 20i(0) + v_L(0) = 0, \text{ which leads to,}$$

$$-16 + 20x2 + v_L(0) = 0 \text{ or } v_L(0) = -24$$

$$Ldi(0)/dt = v_L(0) \text{ which gives } di(0)/dt = v_L(0)/L = -24/1 = -24 \text{ A/s}$$

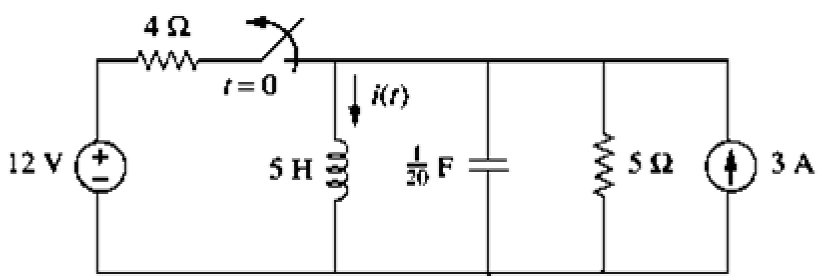
$$\text{Hence } -24 = -2A - 18B \text{ or } 12 = A + 9B \qquad (2)$$

From (1) and (2), B = 1.25 and A = 0.75

$$i(t) = [0.75e^{-2t} + 1.25e^{-18t}] = -i_x(t) \text{ or } i_x(t) = \mathbf{[-0.75e^{-2t} - 1.25e^{-18t}]\ A}$$

$$v(t) = 8i(t) = \mathbf{[6e^{-2t} + 10e^{-18t}]\ A}$$

25. Determine i(t) for t>0 in the circuit shown below



26. Find the output voltage Vo(t) in the circuit of shown below for t>0. If V1=8v, V2=12v, R=2Ω, L=1H, C= 1/5 F.